

# Decay Rates

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1

# Radioactive Decay

Decay (disintegration) is the process by which a radionuclide changes its number of neutrons and protons from an unstable combination to a more stable combination. Some people prefer to use the term transformation rather than decay.

Examples include beta decay, positron decay, electron capture and alpha decay.

In radioactive decay, mass is always lost. This mass is converted to energy and released. The released energy is carried off by any charged particles and/or photons that are emitted (i.e., radiation).

## Activity (A)

Activity is a quantity given the symbol A. It is the decay rate of a specified radionuclide in a sample, i.e., the number of decays per unit time of that radionuclide.

The International Commission on Radiation Units and Measurements (ICRU) definition (ICRU Report 60) follows:

"The activity, A, of an amount of radioactive nuclide in a particular energy state at a given time is the quotient of dN by dt, where dN is the expectation value of the number of spontaneous nuclear transformations from that energy state in the time interval dt."

$$A = \frac{dN}{dt}$$

## Activity (A)

The term “spontaneous” indicates that the transformations are an inherent characteristic of the material and are not induced by external forces (such as neutron bombardment).

Because decay is a random process, it is impossible to say precisely what the activity of a sample is, or will be. We can only specify what the activity was. As such, the ICRU employs the term "expectation value", i.e. the activity is the decay rate we expect to have.

# Units of Activity

## Units of Activity - the curie (Ci)

Originally, the curie was defined by the International Radiological Congress in 1910 as the quantity of radium emanation (radon gas) in equilibrium with one gram of radium. Radium itself was quantified by weighing.

Later , the curie was defined as the activity of one gram of pure Ra-226.

Today, the curie (Ci) is defined as  $3.7 \times 10^{10}$  dps.

According to Ernest Rutherford, the unit was named in honor of both Pierre Curie (the latter had been run over by a coach) and Marie.

# Units of Activity - the curie (Ci)

## Multiples of the curie

$$1 \text{ curie (Ci)} = 3.7 \times 10^{10} \text{ dps} \quad (2.22 \times 10^{12} \text{ dpm})$$

$$1 \text{ millicurie (mCi)} = 3.7 \times 10^7 \text{ dps} \quad (2.22 \times 10^9 \text{ dpm})$$

$$1 \text{ microcurie (uCi)} = 3.7 \times 10^4 \text{ dps} \quad (2.22 \times 10^6 \text{ dpm})$$

$$1 \text{ nanocurie (nCi)} = 3.7 \times 10^1 \text{ dps} \quad (2.22 \times 10^3 \text{ dpm})$$

$$1 \text{ picocurie (pCi)} = 3.7 \times 10^{-2} \text{ dps} \quad (2.22 \text{ dpm})$$

# Units of Activity - the becquerel (Bq)

The becquerel (Bq), named after Henri Becquerel who discovered radioactivity, is the basic unit of activity in the Systeme International (the SI system).

## Multiples of the becquerel

$$1 \text{ becquerel (Bq)} = 1 \text{ dps}$$

$$1 \text{ kilobecquerel (kBq)} = 10^3 \text{ dps}$$

$$1 \text{ megabecquerel (MBq)} = 10^6 \text{ dps}$$

$$1 \text{ gigabecquerel (GBq)} = 10^9 \text{ dps}$$

$$1 \text{ terabecquerel (TBq)} = 10^{12} \text{ dps}$$



## Units of Activity

In real scientific literature, the units of activity would never be expressed as dps or dpm . This is an admission that this handout is not real scientific literature.

The “s”, as in “dps”, is a true unit. The second is a specific amount of time used as a standard of measure.

However, the “d” in “dps” is not a unit. A disintegration is an event, or a thing. It is not a specific amount of anything.

The scientific literature would employ “s<sup>-1</sup>” instead of “dps.”

s<sup>-1</sup> means per second

# Decay Constant

## Decay Constant ( $\lambda$ )

The decay constant, symbolized by the Greek letter lambda, can be described (not quite correctly) as the fraction of the atoms (or activity) of a radionuclide that is expected to decay per unit time.

e.g., a decay constant of  $0.25 \text{ s}^{-1}$  implies that 25% of the radionuclide's atoms (or activity) decay per second.

It can also be thought of as the proportionality constant between the number of atoms of a radionuclide and the activity:

$$A = \frac{dN}{dt} = -\lambda N$$

## Decay Constant ( $\lambda$ )

Unfortunately, decay constants aren't found by looking them up in a table or list. They must be calculated using the following equation:

$$\lambda = \frac{0.693}{T}$$

where T is the half-life of the radionuclide

Each radionuclide has its own unique decay constant.

## Decay Constant ( $\lambda$ ) - examples

a. Calculate the decay constant of I-131 in days<sup>-1</sup>:

Since the half-life of iodine-131 is 8.04 days

$$\begin{aligned}\lambda &= \frac{0.693}{8.04 d} \\ &= 0.0862 d^{-1}\end{aligned}$$

This implies (incorrectly) that 8.62% of the activity decays per day.

## Decay Constant ( $\lambda$ ) - examples

b. Calculate the decay constant of I-131 in  $\text{hr}^{-1}$ :

$$\begin{aligned}\lambda &= \frac{0.693}{8.04 d \times 24 \text{ hr} / d} \\ &= 0.00359 \text{ hr}^{-1}\end{aligned}$$

This implies (incorrectly) that 0.359% decays per hour.

c. Calculate the decay constant of I-131 in  $\text{y}^{-1}$ :

$$\begin{aligned}\lambda &= \frac{0.693}{8.04 d \times 1/365 \text{ y} / d} \\ &= 31.5 \text{ y}^{-1}\end{aligned}$$

This implies (incorrectly) that 3150% decays per year<sub>14</sub>

## Decay Constant ( $\lambda$ )

These three decay constants are all correct.

What is incorrect is the idea that the decay constant is the fraction of the atoms or activity of a nuclide that decays per unit time.

More correctly, the decay constant is the instantaneous decay fraction – the fraction of the activity that decays in an infinitesimally small amount of time.

## The Decay Constant is Immutable (almost)

In general, the decay constant (or half-life) of a radionuclide is not affected by time, space, pressure, temperature, electromagnetic fields, etc.

However, there are exceptions: the chemical form of the sample can affect the decay constant of radionuclides that undergo electron capture and/or internal conversion.

This happens because the chemical bonds formed by an atom affect the availability of the electrons for capture/conversion. For example, the decay constant of Be-7 shows a 0.1% difference between beryllium metal and BeF<sub>2</sub>. Tc-99m, which undergoes internal conversion in nearly 100% of its decays (transitions), shows a 0.3% difference in decay constants between KTcO<sub>4</sub> and Tc<sub>2</sub>S<sub>7.16</sub>



## The Mean Life

The mean (average) life of a radionuclide is calculated as follows:

$$\tau = \frac{1}{\lambda} = \frac{T}{0.693}$$

$\tau$  is the mean (average) life of the radionuclide (e.g., hr)

$\lambda$  is the decay constant of the radionuclide (e.g., hr<sup>-1</sup>)

$T$  is the half life of the radionuclide (e.g., hr)

# Decay Equation

# The Decay Equation

The following equation calculates the activity (A) for a specified number of atoms of a radionuclide (N)

$$A = \lambda N$$

The number of decays per unit time equals the fraction of the atoms decaying per unit time multiplied by the number of atoms.

Unfortunately, this equation is rarely of practical value because the number of atoms is continually decreasing.

# The Decay Equation

However, the next equations are useful because they tell us the number of atoms or activity of the radionuclide as a function of time:

$$N_t = N_0 e^{-\lambda t}$$
$$A_t = A_0 e^{-\lambda t}$$

$N_t$  is the number of atoms of a radionuclide at time  $t$

$N_0$  is the number of atoms of a radionuclide at time zero

$A_t$  is the activity of a radionuclide at time  $t$

$A_0$  is the activity of a radionuclide at time zero

$e$  is the base of the natural log system,  $e = 2.718281828$

## The Decay Equation - example

Calculate the activity of a 100 Ci I-131 source after it has decayed for 1.5 days.

Before using an equation, it's a good idea to make a guesstimate as to the answer.

1.5 days is about one-fifth of iodine's 8 day half-life. If 8 days reduces the activity to 50 percent of what it was, 1.5 days should only reduce it to ninety percent or so of its original activity. Therefore, our best guess is that the activity of the iodine will be about 90 Ci.

Now we are ready to use the decay equation.

## The Decay Equation - example

$$\begin{aligned}A_t &= A_0 e^{-\lambda t} \\ &= 100 e^{-0.0862 \times 1.5} \\ &= 100 e^{-0.1293} \\ &= 100 \times 0.8787 \\ &= 88 \text{ Ci}\end{aligned}$$

Since this answer is right in line with our estimate, we probably did the calculation correctly.

## The Meaning of $e^{-\lambda t}$

The term  $e^{-\lambda t}$  is the fraction that remains.

As such, the equation  $A = A_0 e^{-\lambda t}$  means that the activity that you have is the activity you started with times the fraction that remains!

Similarly, the equation  $N = N_0 e^{-\lambda t}$  means that the number of atoms of a radionuclide that you have is the number you started with times the fraction that remains!

## Back Calculating Activities

In some situations, it is necessary to back calculate the activity of a source to some earlier date:

$$A_t = A_0 e^{\lambda t} = \frac{A_0}{e^{-\lambda t}}$$

$A_t$  is the activity at some earlier specified time  $t$

$A_0$  is the activity at time zero

Note,  $A_t$  is greater than  $A_0$



# Calculating the Decay Time

Should it be necessary to calculate how much time ( $t$ ) is required for a radionuclide to decay from an initial activity ( $A_0$ ) to a specified final activity ( $A_t$ ), the following equation can be used:

$$t = -\frac{\ln\left(\frac{A_t}{A_0}\right)}{\lambda}$$

For what it is worth, the value  $A_t/A_0$  is the fraction of the original activity remaining.

# Calculating the Decay Time

The number of half lives required to reduce its original activity ( $A_0$ ) to a desired value ( $A_t$ ) is calculated as follows:

$$\text{number of half lives} = 1.443 \times \ln\left(\frac{A_t}{A_0}\right)$$

As such it takes:

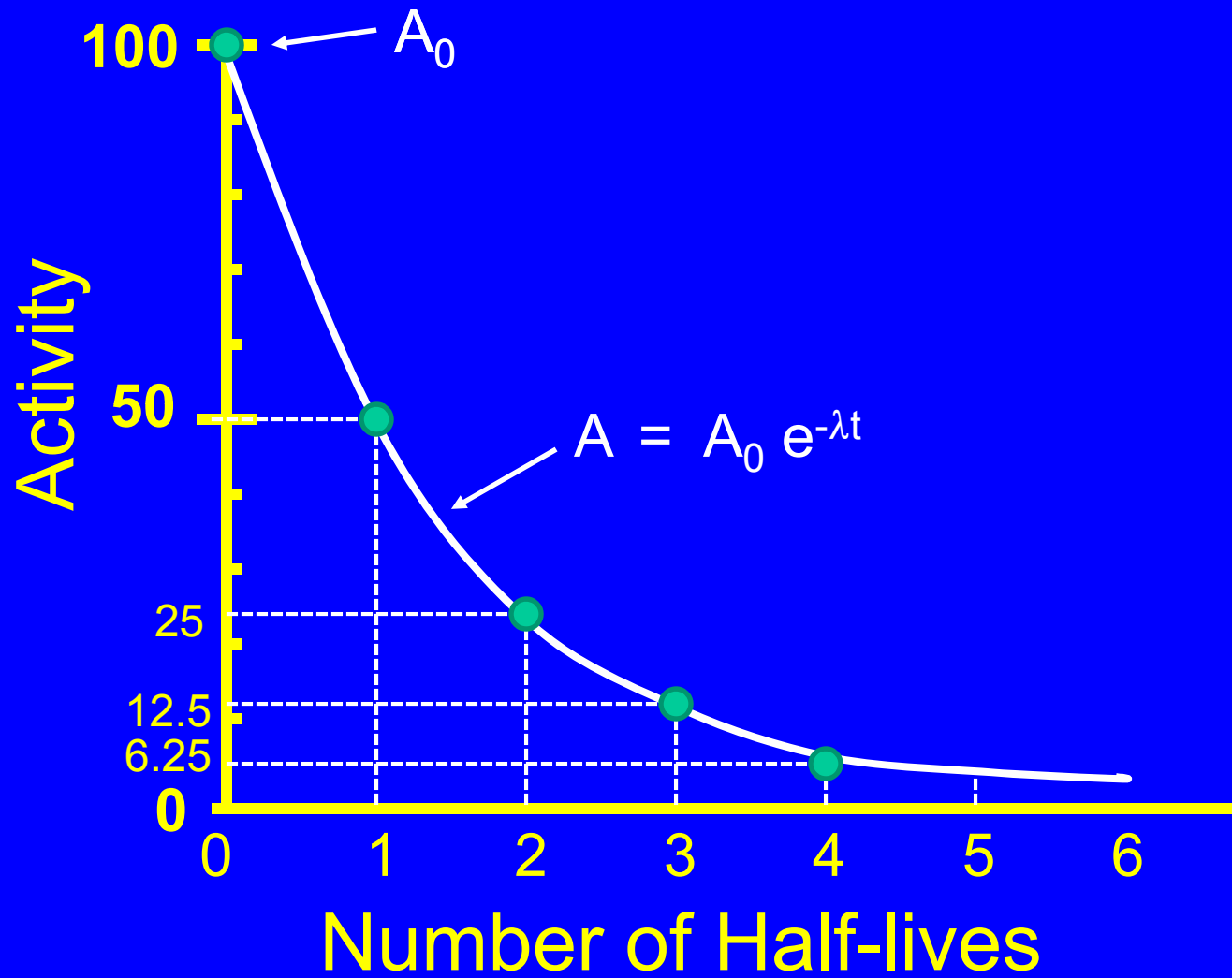
3.32 half lives for the activity to decay to 10% of its original activity

6.65 half lives for the activity to decay to 1% of its original activity

9.97 half lives for the activity to decay to 0.1% of its original activity

# Graphing Activity as a Function of Time

# Linear Plot



## Problem with a Linear Plot

The problem with a linear plot is that a curved line can be hard to draw (as if anyone actually does that).

It is hard to interpolate between data points.

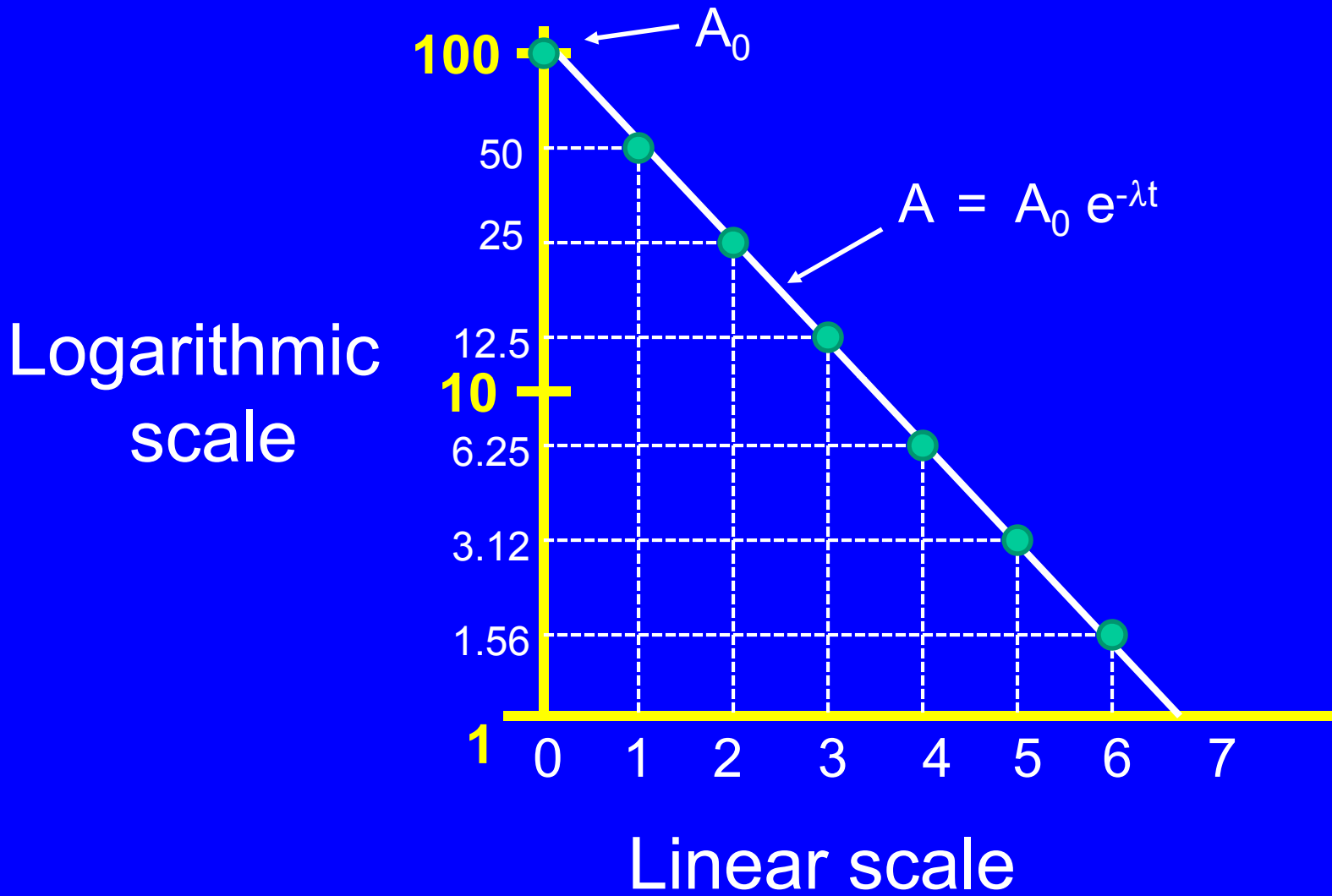
It is hard to extrapolate beyond data points.

# Semilog Plot

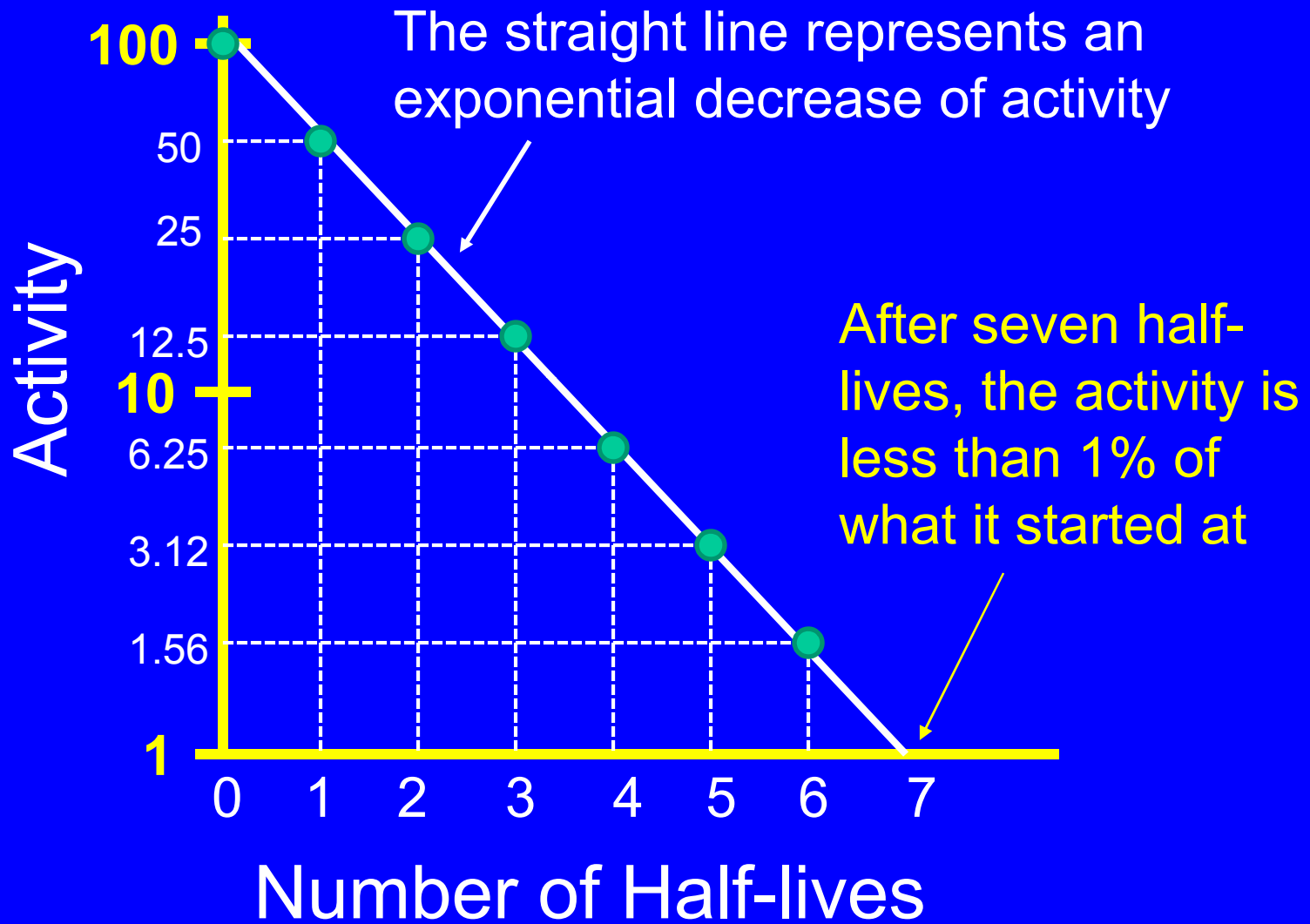
A semilog plot such as the one on the following slide will solve all our problems!

On the semilog plot, time is indicated with a linear scale while activity (or number of atoms) is indicated with a logarithmic scale.

# Semilog Plot



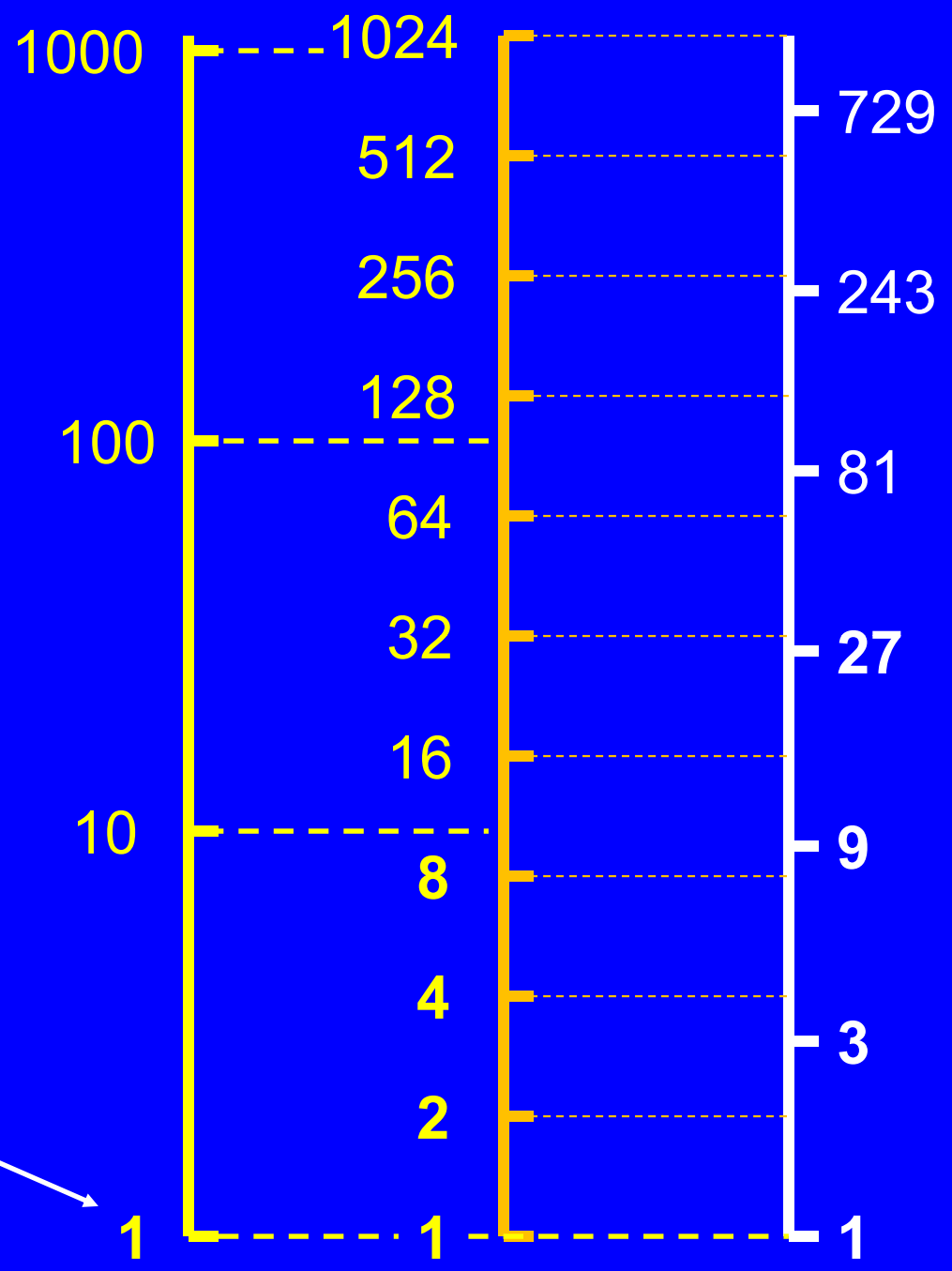
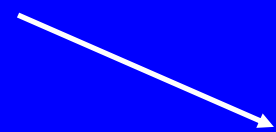
# Semilog Plot





# Logarithmic scale

A log scale can begin with any number except zero



## Specific Activity

The specific activity of a radionuclide is its activity per unit mass (of the pure material). The "old" and SI units are Ci/g and Bq/kg respectively.

As might be expected from the original definition of the curie, the specific activity of Ra-226 is close to 1 Ci/g. Saying that something has a high specific activity is the same as saying that it has a short half-life.

To calculate the specific activity (SpA) in Bq/g, we use the following formula:

$$\text{SpA} = N_g \lambda$$

$N_g$  is the number of atoms of the nuclide in one gram of pure material and  $\lambda$  is the decay constant in units of  $\text{s}^{-1}$

## Specific Activity – example calculation

Example: calculate the specific activity of U-235.

First, calculate the number of U-235 atoms ( $N_g$ ) in one gram

Since there are 235 grams of U-235 per mole, in one gram there will be  $1/235$  moles, i.e,  $4.255 \times 10^{-3}$  moles.

Since there are  $6.023 \times 10^{23}$  atoms per mole (Avagadro's number), there are the following number of atoms of U-235 in one gram of U-235:

$$\begin{aligned} N_g &= 4.255 \times 10^{-3} \text{ moles/gram} \times 6.023 \times 10^{23} \text{ atoms/mole} \\ &= 2.563 \times 10^{21} \text{ atoms/gram} \end{aligned}$$

## Specific Activity – example calculation

Next, calculate the decay constant ( $\lambda$ ) for U-235, the half-life (T) of which is  $7.04 \times 10^8$  a (years).

Because we want to calculate an activity in dps, the half-life of U-235 should be expressed in seconds.

$$\lambda = 0.693/T$$

$$= 0.693/(7.04 \times 10^8 \text{ a} \times 365.25 \text{ d/a} \times 24 \text{ h/d} \times 3600 \text{ s/h})$$

$$= 0.693/2.222 \times 10^{16} \text{ s}$$

$$= 3.119 \times 10^{-17} \text{ s}^{-1}$$

## Specific Activity – example calculation

Finally, multiply  $\lambda$  times  $N_g$  to calculate the specific activity.

$$\begin{aligned}\text{SpA} &= N_g \lambda \\ &= 2.563 \times 10^{21} \text{ atoms/g} \times 3.119 \times 10^{-17} \text{ s}^{-1} \\ &= 7.995 \times 10^4 \text{ s}^{-1} \text{ g}^{-1} \text{ (dps per gram)} \\ &= 7.995 \times 10^4 \text{ Bq g}^{-1} \text{ (Bq per gram)} \\ &= 2.16 \times 10^{-6} \text{ Ci g}^{-1} \text{ (Ci per gram)}\end{aligned}$$

# Air Kerma Strength

Air kerma strength is the quantity used to describe the strength of brachytherapy sources. It can be thought of as a substitute for activity. More specifically, it is the air kerma rate at some specified distance from the source. The equation for the air kerma strength is:

$$S_k = K l^2$$

$S_k$  is the air kerma strength (e.g., uGy m<sup>2</sup>/h)

$K$  is the air kerma rate in free space (e.g., uGy/h)

$l$  is the reference distance (m)

The units of air kerma strength are often given the symbol  $U$  where  $1 U = 1 \text{ uGy m}^2/\text{h}$

# Air Kerma Strength

Air kerma strength is an empirical measured quantity rather than a calculated one.

It is inappropriate to calculate the air kerma rate of a brachytherapy source based on its activity (e.g., mCi).

Similarly, it is inappropriate to calculate the activity of the source based on its air kerma strength.

## Air Kerma Strength

Despite the previous comments, conversion factors from “apparent” activity to air kerma strength are required in the real world. The following are from Med Phys 18 (3): 439-448.

Au-198 seeds      1 mCi  $\approx$  2.06 U

Pd-103 seeds              1 mCi  $\approx$  1.29 U

I-125 seeds          1 mCi  $\approx$  1.27 U

See Code of practice for brachytherapy physics: *Report of the AAPM Radiation Therapy Committee Task Group No. 56* (Medical Physics 24 (10); 1557-1598, Oct. 1997).



# Appendix A

## The Meaning of $(1 - e^{-\lambda t})$

## The Meaning of $(1 - e^{-\lambda t})$

The term  $(1 - e^{-\lambda t})$  is the fraction that decayed.

If  $e^{-\lambda t} = 0.25$ , then 25% of the original number of atoms remain.

And  $(1 - 0.25)$  means that 0.75 or 75% of the original number of atoms have decayed over the time period  $t$ .

The number of decays of a radionuclide during time  $t$  is:

$$N_d = N_0 (1 - e^{-\lambda t})$$

$N_d$  is the number of decays during time  $t$

$N_0$  is the number of atoms of the radionuclide at  $t = 0$

## The Meaning of $(1 - e^{-\lambda t})$

When a radioactive sample is counted with a detector, the number of counts is:

$$C = N_0 E (1 - e^{-\lambda t})$$

C is the number of counts

$N_0$  is the number of atoms of the radionuclide at the start of the count ( $t = 0$ )

E is the detector efficiency (counts/decay)

## The Meaning of $(1 - e^{-\lambda t})$

Rewriting the previous equation gives:

$$N_0 = \frac{C}{E(1 - e^{-\lambda t})}$$

Multiply both sides by  $\lambda$  gives:

$$\lambda N_0 = \frac{\lambda C}{E(1 - e^{-\lambda t})}$$

Hence:

$$A_0 = \frac{\lambda C}{E(1 - e^{-\lambda t})}$$

# Appendix B

## Some Obsolete Units of Activity

# Some Obsolete Units of Activity

While the following units are no longer used, they can still be encountered in early literature and/or on old radioactive sources.

## 1. Rutherford

The Rutherford (rd) was defined in 1930 by the International Radium Standard Commission as  $10^6$  dps. Such a unit was needed because the Commission objected to using the curie to describe the activities of radionuclides outside the radium family (radium or its daughters).

# Some Obsolete Units of Activity

## 2. Strength

The strength was a poorly defined unit primarily employed between 1904 and 1920. It was related to the intensity of the gamma rays emitted by uranium. If a radium source was said to have a strength of 500,000, it meant that it had a gamma intensity 500,000 times greater than an identical mass of uranium. The terms intensity and activity were sometimes used to mean the same thing.

# Some Obsolete Units of Activity

## 3. Uranie

The uranie was defined in 1905 by W. D. Butcher, a president of the Rontgen Society, as the quantity of radiation emitted by one gram of uranium. One gram of radium was equal to one million uranies.



# Some Obsolete Units of Activity

## 4. Mache

This unit was defined in 1904 and used exclusively for radon. In general, it was used to express the activity of radon in one liter of water (occasionally air). One mache was equal to 364 pCi. If it wasn't stated, it could usually be inferred that the activity was per liter.

The Mache Unit was applied in different ways and it is difficult to equate it to a radium activity. However, as a rough approximation, a 2700 Mache Units usually indicated a radium-226 activity of at least 1 uCi.

# Some Obsolete Units of Activity

## 5. Eman

One eman was equal to 100 pCi of radon. In other respects, the eman was similar to the mache unit.

# Some Obsolete Units of Activity

## 6. Special Curie Unit for Natural Uranium

Natural uranium, whether in ore, or in chemically purified form, is a mix of three isotopes: U-234, U-235 and U-238. In one gram of pure natural uranium, these isotopes will have the following activities:  $7.36 \times 10^5$  dpm U-234;  $3.42 \times 10^4$  dpm U-235; and  $7.36 \times 10^5$  dpm U-238.

Originally, limits on natural uranium concentrations in air, water etc. were expressed in milligrams. By mass, natural uranium is almost entirely U-238. Later, when it became common to express those same limits in units of activity, it had to be clarified whether the limit expressed the total activity (summed activities of U-234, U-235 and U-238) or just the activity of the U-238.

# Some Obsolete Units of Activity

## 6. Special Unit for Natural Uranium - continued

To reduce the confusion, the ICRP (1955) stated " the curie of natural uranium is considered to correspond to  $3.7 \times 10^{10}$  dis/sec from U-238,  $3.7 \times 10^{10}$  dis/sec from U-234 and  $9 \times 10^8$  dis/sec from U-235". Later, the value for U-235 was corrected to  $1.7 \times 10^9$  dis/sec. In other words, the curie for natural uranium was  $7.57 \times 10^{10}$  dps ( $4.542 \times 10^{12}$  dpm). Therefore, the activity of one gram of pure uranium would be  $3.3 \times 10^{-7}$  curies.