Recent Advances in PTHA Methodology

Randall J. LeVeque, University of Washington

F. I. González, University of Washington
Loyce M. Adams, University of Washington
Knut Waagan, University of Washington
Guang Lin, Pacific Northwest National Laboratory

NRC Workshop on

Probabilistic Flood Hazard Assessment

January 29–31, 2013

Crescent City PTHA Pilot Study

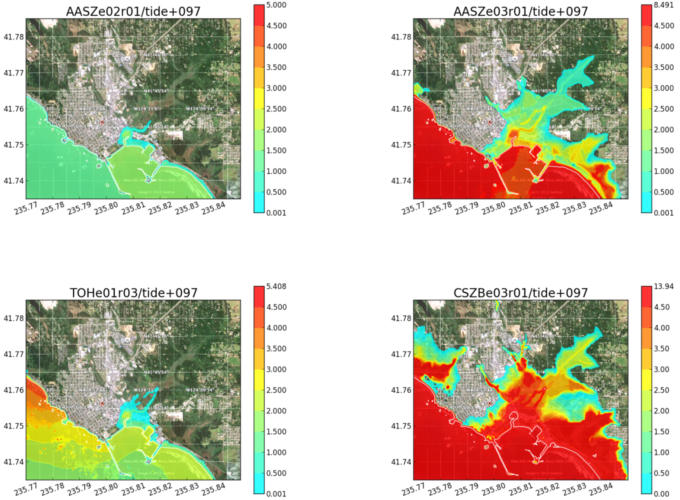
Preliminary results will be shown from ongoing pilot study.

Supported by BakerAECOM, as part of a coastal modeling/mapping effort funded by the FEMA Region IX office as part of the new California Coastal Analysis and Mapping Project (CCAMP).

Simulations done with GeoClaw model (shallow water equations)   **www.clawpack.org/geoclaw**

Crescent City, CA

Four sample event realizations



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- Cumulative probability distribution of tide stage.

Some recent advances

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Some limitations:

- Proper probability density for slip distribution is hard to determine — geophysics problem.
- How to add in the possibility of submarine landslides affecting tsunami size?
- Many other uncertainties, e.g. friction coefficient and other aspects of mathematical model / numerical method.

Hazard Curves

First define hazard curve: for each location (x, y):

$$P(\zeta) = P(\zeta; x, y) = \text{Prob}[\text{inundation } \geq \zeta \text{ in one year}].$$

Example: If only one possible event E_1 with recurrence time T_1 (Poisson rate $\nu_1 = 1/T_1$), that floods to level $H_1(x, y)$, then

$$P(\zeta) = \begin{cases} 1 & \text{if } \zeta = 0, \\ 1 - e^{-\nu_1} & \text{if } 0 < \zeta < H_1, \\ 0 & \text{if } \zeta > H_1. \end{cases}$$

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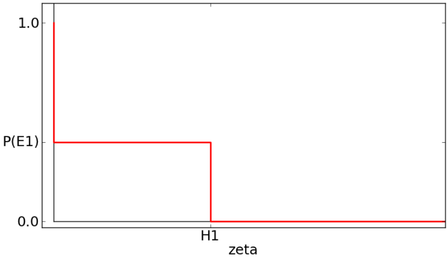
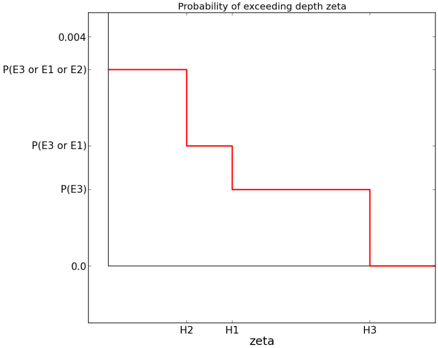


Hazard Curves

Example: Three possible events $E_1,\ E_2,\ E_3$ with recurrence times $T_1,\ T_2,\ T_3$, that flood to levels $H_1,\ H_2,\ H_3$.

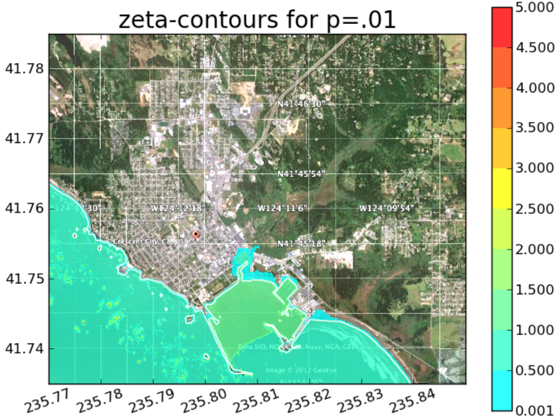


Where, for example,

$$P(E3 \text{ or } E1) = 1 - (1-p_3)(1-p_1) = 1 - e^{-(\nu_3+\nu_1)}.$$

Probabilistic maps of flooding depth

Standard view: Map of flooding depth for fixed probability



Preliminary Results — Not for Use

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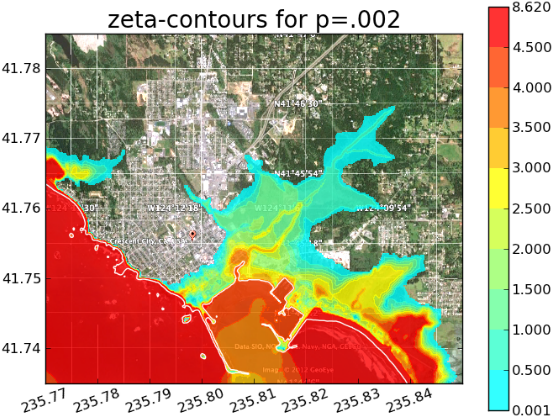

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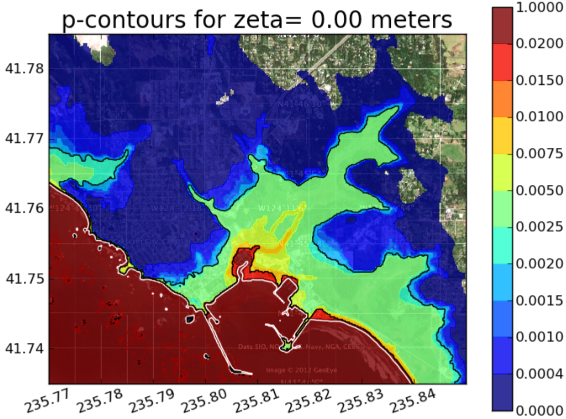
Alternative map: Probability of exceeding fixed depth



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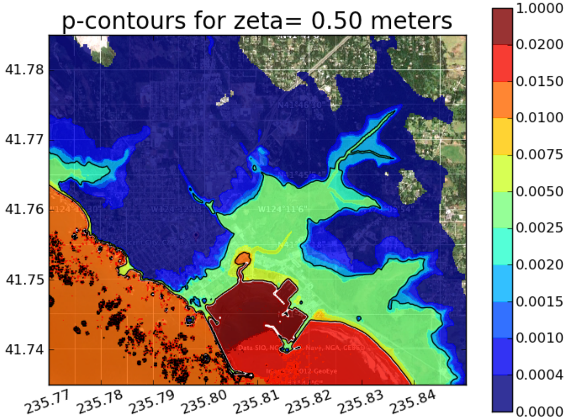

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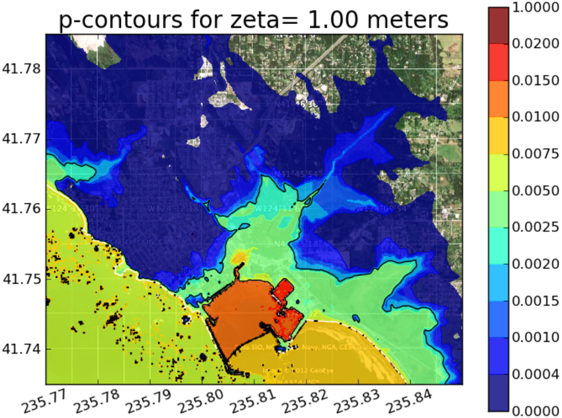

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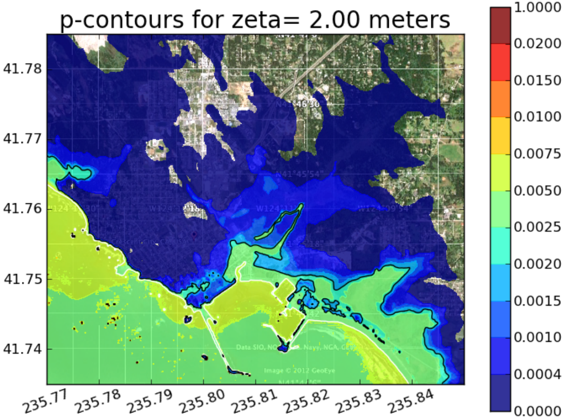

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Effect of tides on inundation at Crescent City

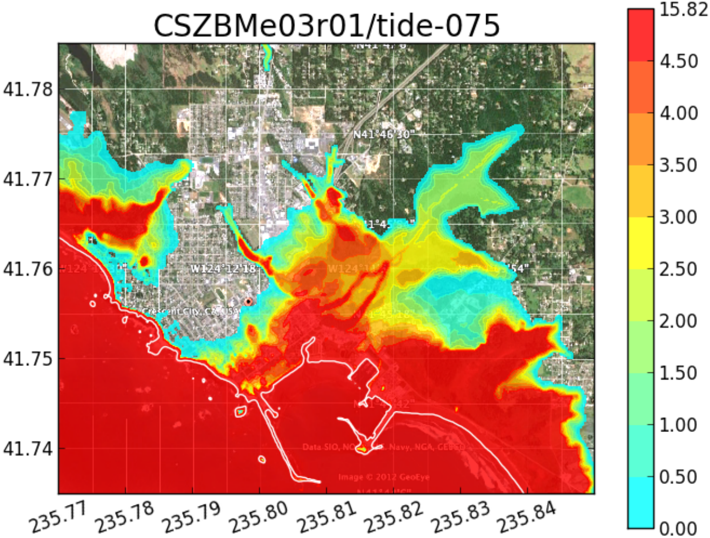
Bathymetry uses vertical datum MHW (Mean High Water).

Tidal range:

MHHW ≈ MSL $+97$ cm.
MHW ≈ MSL $+77$ cm.
MLW ≈ MSL -75 cm.
MLLW ≈ MSL -113 cm.

How much difference does modifying tide stage have on inundation?

Sample inundation from CSZ M9 earthquake



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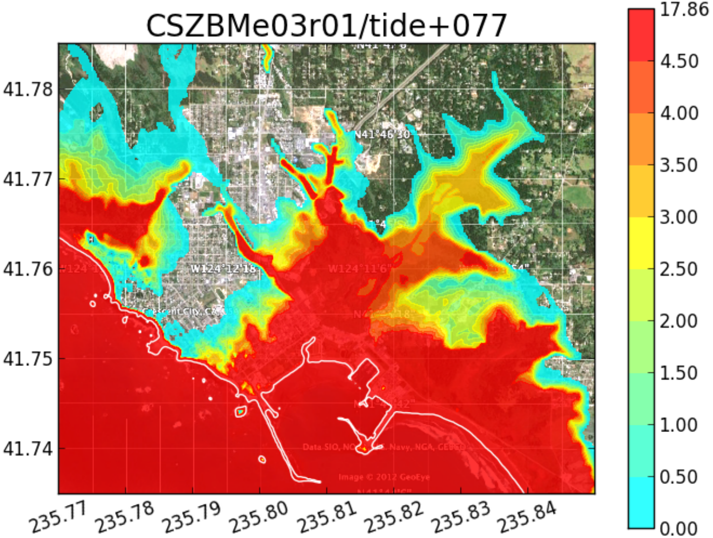
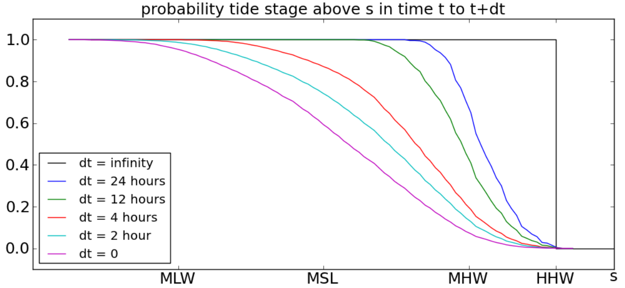


Tidal uncertainty

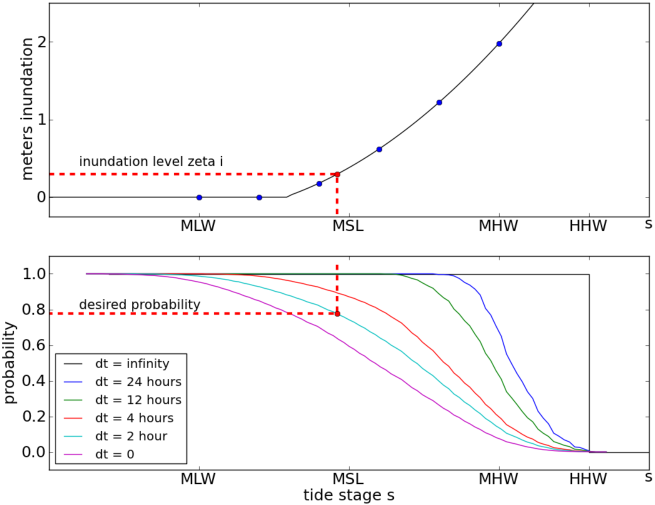
Suppose we determine for a given event that largest waves are all seen within a period of Δt hours after arrival of first wave.

Then we can use tide record to determine the cumulative probability that the tide stage will be above s at *some* time between t and $t + \Delta t$. (Where t is assumed to be a random time in tide cycle when first wave hits.)

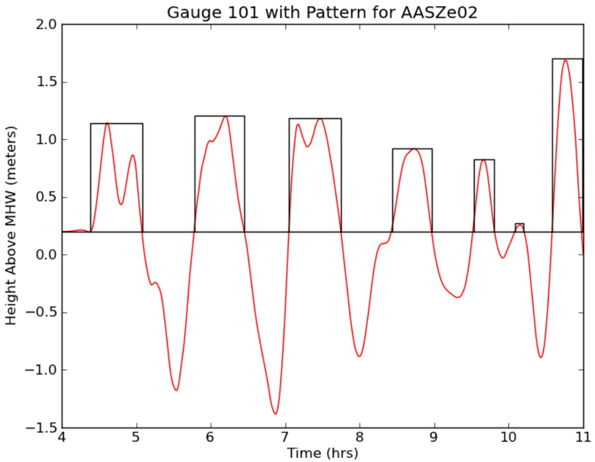


Tidal uncertainty

For example, if $\Delta t = 2$ hours is appropriate for this event:

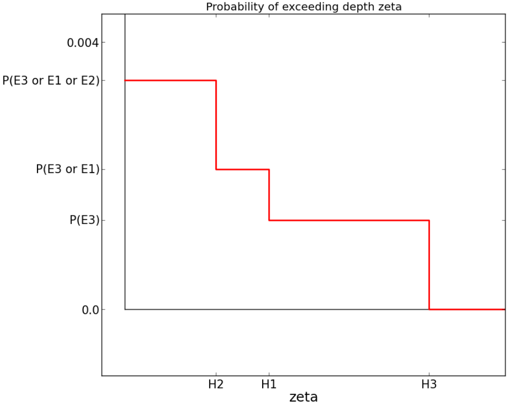


The pattern method



Effect of tidal uncertainty on hazard curves

Example: Three events with no tidal uncertainty:



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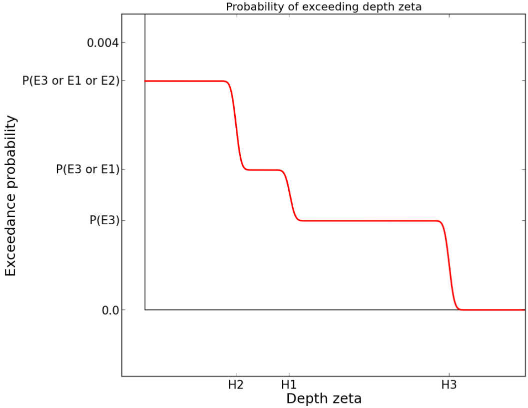
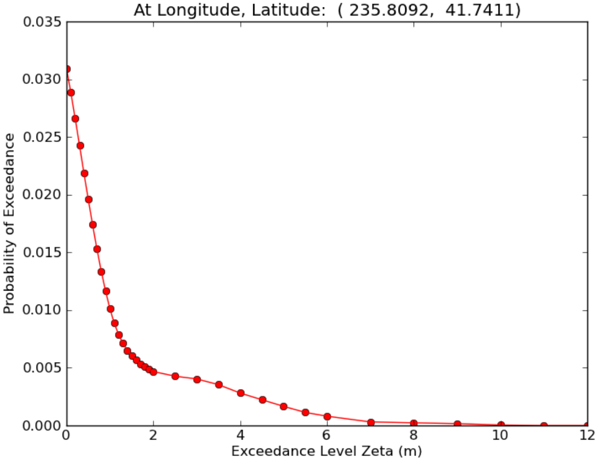


Hazard curve with many events + tides



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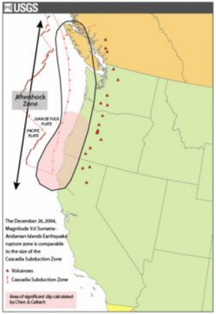
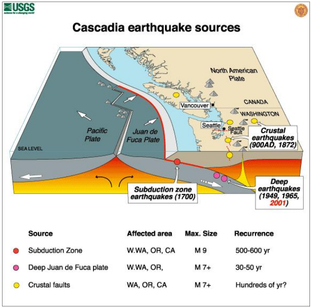
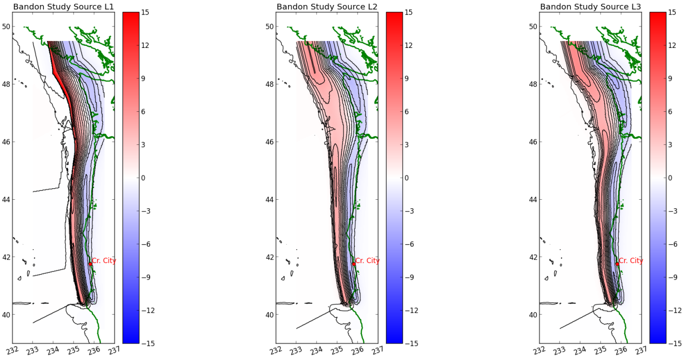
Cascadia Subduction Zone (CSZ)





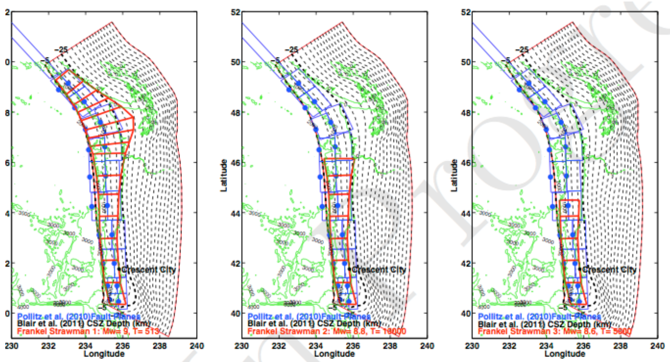
- 1200 km long off-shore fault stretching from northern California to southern Canada.
- Last major rupture: magnitude 9.0 earthquake on January 26, 1700.
- Tsunami recorded in Japan with run-up of up to 5 meters.
- Historically there appear to be magnitude 8 or larger quakes every 500 years on average.

Sample seafloor deformations



R. C. Witter and Y. Zhang and K. Wang and G. R. Priest and C. Goldfinger and L. L. Stimely and J. T. English and P. A. Ferro, *Simulating tsunami inundation at Bandon, Coos County, Oregon, using hypothetical Cascadia and Alaska earthquake scenarios*, Oregon Department of Geology and Mineral Industries Special Paper 43, 2011. (15 realizations)

CSZ fault geometry and possible slip regions



3 Possible slip regions (Art Frankel, USGS) and fault geometry of

Pollitz, McCrory, Wilson, Svarc, Puskas, Smith,
http://doi.wiley.com/10.1111/j.1365-246X.2010.04546.x

and Blair, McCrory, Oppenheimer, **http://pubs.usgs.gov/ds/633/**

Generating stochastic source realizations

For a given event (e.g. CSZ M_w 9.1) we need to use more than one possible slip distribution.

To simplify, consider an event with a single fault plane for which we know strike, dip, rake, and the integral of slip (from M_w).

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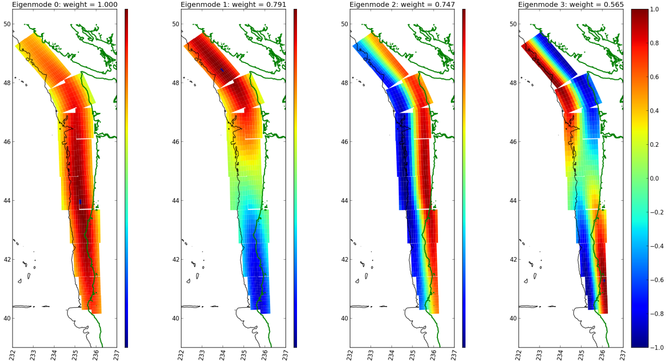
How to generate many realistic slips d_i?

Karhunen-Loève Modes of CSZ



Eigenfunctions of Covariance assuming correlation lengths of 40% along-strike and down-dip.

P. M. Mai and G. C. Beroza, *A spatial random field model to characterize complexity in earthquake slip*, J. Geophys. Res. 107(2002).

Karhunen-Loève expansion

Let $(\lambda_k,\ v_k)$ be eigenvalue/vectors of C, for $k = 1,\ 2,\ \ldots,\ m$.

Order so that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$.

Eigenvectors for larger k are more oscillatory.

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Then realizations $d = [d_1,\ \ldots,\ d_m]$ can be generated by

$$d = \mu + \sum_{k=1}^{m} z_k \sqrt{\lambda_k}\, v_k$$

where the z_k are chosen to be independent from $N(0,1)$
(normally distributed with mean 0 and variance 1).

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Advantages:

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Moreover, Once we apply Okada model to d, oscillatory components get further smoothed.

Even lower dimensional space of seafloor deformations?

Uncertainty quantification techniques

Why is dimension reduction important?

May be able to do better than brute-force Monte-Carlo by doing smaller number of simulations at carefully chosen points in stochastic space, represented by vectors

$$[z_1,\ z_2,\ \ldots, z_r] \qquad (r = \text{number of stochastic dimensions})$$

and building up polynomial approximation to response surface (stochastic collocation).

Then use known probability distribution of the z_k to compute probability of response.

Hard to do in large number of dimensions, but can explore techniques such as sparse grids, importance sampling, MCMC, multi-level MC.

Dealing with Uncertainties

Aleatoric uncertainties: (Mathematical problem)

Given the correct probability density of slip distribution, there are techniques to reduce dimension and sample efficiently.

Epistemic uncertainties: (Geophysical problem)

- What is correct distribution/recurrence of different magnitude quakes?
- What is correct fault geometry?
- Is there tapering of slip?
- Is K-L expansion with Gaussian weights correct?
- What is correct correlation function?

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